Matrices:

1\. A matrix of order m by n is an array of quantities A_{ij} called elements arranged in m rows and n columns.

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{bmatrix} \text{ or } \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{pmatrix}$$

It may be denoted by $[A_{ij}]$ or (A_{ij}) for $i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, n$.

* for $m = n$, matrix is square matrix of order m by m

* for $m = 1$ $\rightarrow$ Row matrix or Row vector

* for $n = 1$ $\rightarrow$ Column matrix or Column vector

* For the square matrix $(m = n)$, the diagonal containing elements $A_{11}, A_{22}, A_{33}, \cdots A_{mm}$ is called the principal or main diagonal

Unit Matrix (I) $\rightarrow$ $A_{ii} = 1$ and $A_{ij} = 0$ for $i \neq j$
(For square matrix only)

metric tensor:

In rectangular coordinate the differential arc length is given by

$$ds^2 = dx^2 + dy^2 + dz^2. \quad \text{—— (1)}$$

The transformation of the above arc length to general curvilinear coordinates (discussed in earlier lecture notes) we obtain

$$ds^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} g_{ij}\, du_i\, du_j \quad \text{—— (2)}$$

For N-dimensional space having coordinates $(x^1, x^2, \ldots x^N)$ the line element ds given in quadratic form is called metric form or metric

$$ds^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} g_{ij}\, dx^i dx^j$$

$$\text{or} \quad \boxed{ds^2 = g_{ij}\, dx^i dx^j} \quad \text{—— (3)}$$

where we have used Einstein summation convention. (Repeated indices are summed over).

g_{ij} → may be functions of x^i with the condition that $\det(g_{ij}) \neq 0$. and the space is called as Riemannian space. For the particular case that g_{ij} are independent of x^i → space becomes Euclidean space.

Associated tensors:

Assume. A^i → arbitrary contravariant vector

Now $\quad A^i g_{ij} = A_j$

A^i and A_j → contravariant and covariant components of same vector

Again

$$A_\nu g^{\nu k} = A^\nu g_{\nu j} g^{j k} = A^\nu \delta_\nu^k = A^k \quad \text{— (4)}$$

Eq. (4) shows that A_j & A^ν ~~are~~ have reciprocal relation.

Tensors A^ν and A_ν are called associate tensors.

Moment of inertia tensor:

The angular momentum $\vec{L} = \sum_\nu m_\nu (\vec{r}_\nu \times \vec{v}_\nu)$ — (5)

$m_\nu \to$ mass of the ν-th point mass
$\vec{r}_\nu \to$ position vector of m_ν w.r.t. origin
$\vec{v}_\nu \to$ linear velocity of m_ν

If $\vec{\omega} \to$ angular velocity vector of the body

$$\vec{v}_\nu = \vec{\omega} \times \vec{r}_\nu$$

$$\therefore \vec{L} = \sum m_\nu [\vec{r}_\nu \times (\omega \times \vec{r}_\nu)] = \sum_\nu m_\nu [\omega r_\nu^2 - \vec{r}_\nu (\vec{r}_\nu \cdot \vec{\omega})] \quad \text{— (6)}$$

{we have used $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Now writing in cartesian components

$$\vec{L} = (L_x, L_y, L_z), \quad \vec{r}_\nu = (x_\nu, y_\nu, z_\nu)$$
$$\vec{\omega} = (\omega_x, \omega_y, \omega_z),$$

Now from eq (6)

$$L_x = I_{xx}\omega_x + I_{xy}\omega_y + I_{xz}\omega_z$$
$$L_y = I_{yx}\omega_x + I_{yy}\omega_y + I_{yz}\omega_z \quad \text{— (7)}$$
$$L_z = I_{zx}\omega_x + I_{zy}\omega_y + I_{zz}\omega_z$$

where, $I_{xx} = \sum_\nu m_\nu (r_\nu^2 - x_\nu^2)$, $I_{yy} = \sum_\nu m_\nu (r_\nu^2 - y_\nu^2)$, $I_{zz} = \sum_\nu m_\nu (r_\nu^2 - z_\nu^2)$

$$I_{xy} = I_{yx} = -\sum_\nu m_\nu x_\nu y_\nu, \quad I_{yz} = I_{zy} = -\sum_\nu m_\nu y_\nu z_\nu$$
$$I_{zx} = I_{xz} = -\sum_\nu m_\nu z_\nu x_\nu$$

Eq. (7) shows that $\vec{L}$ is not parallel to $\vec{\omega}$. We write

$\boxed{L_i = I_{ij} \omega_j}$ — (8), $\left.\begin{matrix} L \\ \omega_j \end{matrix}\right\} \to$ vectors

$I_{ij} \to$ tensor (from quotient law it is clear from eq. 8)